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Half-space invisible states in dielectric particles

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Abstract: The concept of invisible optical states in dielectric particles is developed. Two cases for excitation of invisible states are discussed. The first one is the excitation in the microparticles with fixed shapes (e.g. spheres) by variation of the properties of incident radiation. The second one is the search for a complex shape of a particle in which invisible states are excited for fixed properties of the incident radiation (e.g. a plane wave). Based on the proposed numerical assessment of the invisibility of the scattered field, a method for finding invisible particles by varying its shape has been developed. A method for calculating the scattered field is generalized in the framework of the theory of surface perturbation for the case of an arbitrary initial shape of the particle.

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1. Introduction

It has been widely accepted that even a very small particle will be visible due to light scattering. The idea of stealth technology [1] naturally arose to help make objects invisible. The idea of invisibility is related to the problem of controlling the scattering of light, and in particular the suppression of the radiation of the scattered field by a particle and considered in many works [2–7]. A perfect invisible object has zero scattering, this concept is called bound states in the continuum [8]. Based on this idea, a number of implementations of the invisibility effect have been proposed [9–12]. More details about this area can be found in the review [13]. In this paper, we developed an original approach to the implementation of the optical invisibility effect. When observing objects using a microscope, information about the source field on a certain plane is collected, thus we limit ourselves to suppressing scattering only in one half-space. The effect of invisibility occurs with a special structure of the field, which ensures the invisibility of the object with a given shape and a refractive index. We also consider the task of finding an invisible form of an object for a given incident field. The difference from the previous works is that we aim to suppress the scattered far field for all multipoles, but not for some particular multipole, such as a dipole. Secondly, we suppress scattering not in one particular direction, but in the entire half-space. Finally, the third difference refers to the original concept of invisible states [14]. These states are nontrivial localized solutions of Maxwell's equations which do not radiate energy within a half-space.

2. Concept of invisible particles

The scattering of electromagnetic waves by a particle can be described using the T-matrix [15,16], which relates the incident field and the field scattered by the particle. We consider homogeneous particles with a constant refractive index at a specific wavelength of light. The incident, $\mathbf{E}^{inc}$, and the scattered, $\mathbf{E}^{sca}$, fields can be represented by the coefficients (g_{lm}, f_{lm}) and (b_{lm}, a_{lm}) in the expansion in terms of basis functions, which can be chosen as spherical harmonics $\mathbf{M}$ and $\mathbf{N}$ [17]:

$$\mathbf{E}^{inc} = \sum_{l,m} f_{lm}\mathbf{N}^{(1)}_{lm} + g_{lm}\mathbf{M}^{(1)}_{lm}, \quad (1)$$

$$\mathbf{E}^{sca} = \sum_{l,m} a_{lm}\mathbf{N}^{(3)}_{lm} + b_{lm}\mathbf{M}^{(3)}_{lm}. \quad (2)$$

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The T-matrix connects the columns of the incident field coefficients $E^i = (g_{lm}, f_{lm})$ and the scattered field coefficients $E^s = (b_{lm}, a_{lm})$ (the coefficients l, m run through all possible values):

$$E^s = TE^i. \tag{3}$$

Our task is to find invisible particles in a half-space, for example, for $z > 0$. By invisibility we mean the following fact. Let the asymptotic expansion of the field E^s in the far zone has the form:

$$\mathbf{E}^{sca} = \mathbf{F}(\theta, \varphi) \exp(ikr)/(ikr), \tag{4}$$

where k is the wavenumber in the medium, $\mathbf{F}$ is the scattering amplitude, (r, θ, φ) are spherical coordinates with the origin at the center of the particle. The time dependence in the form $exp(-i\omega t)$ is omitted. The particle is invisible in the half-space $z > 0$ if

$$\mathbf{F}(\theta, \varphi) \equiv 0, \ at\ \theta \leq \pi/2. \tag{5}$$

According to this definition, we call *invisible states* as such scattering states $E^s = (b_{lm}, a_{lm})$ of the particle under which the condition (5) is satisfied. By using expansion (2) for the scattered field, we can give the following expression for the scattering amplitude:

$$\mathbf{F}(\theta, \varphi) = S_1\mathbf{e}_\theta + S_2\mathbf{e}_\varphi, \tag{6}$$

where $\mathbf{e}_\theta$, $\mathbf{e}_\varphi$ are unit vectors of the spherical coordinate system. Functions S_1, S_2 are defined as

$$\begin{aligned} S_1 &= -\sum\nolimits_{l,m} \exp(im\varphi)(-i)^{l+1}[a_{lm}\tau_{lm} + b_{lm}\pi_{lm}], \\ S_2 &= -\sum\nolimits_{l,m} \exp(im\varphi)(-i)^{l}[a_{lm}\pi_{lm} + b_{lm}\tau_{lm}], \end{aligned} \tag{7}$$

where $\pi_{lm}(\cos\theta) = mP_l^m(\cos\theta)/\sin\theta$, $\tau_{lm}(\cos\theta) = dP_l^m(\cos\theta)/d\theta$ and P_l^m are associated Legendre polynomials.

3. General theory of invisibility states in scattering particles

We consider two approaches for the implementation of particles invisibility. In the first case, for a given particle, we look for such fields $\mathbf{E}^{\text{inc}}$ incident on the particle, at which the field $\mathbf{E}^{\text{sca}}$ scattered by the particle is invisible. Obviously, due to expansion (1), it suffices to find the expansion coefficients of the incident field E^i. Since the properties of the particle are predetermined, we assume that the T-matrix is known. In the second case, for a given incident field $\mathbf{E}^{\text{inc}}$, we look for parameters of the particle, such as refractive index, size and shape, for which the scattered field becomes invisible.

Before discussing these issues, let us make some remarks. First, we have to limit ourselves to some approximations, i.e., the particles are not absolutely invisible. The reasons for these limitations becomes clear later. Note also that the solution of the second problem is much more complicated than the first one.

To solve the two problems presented above, first of all, it is necessary to find nonzero columns E^s for which the invisibility condition (5) is satisfied. The matrix formalism in the theory of optical imaging reduces this problem to a system of linear equations. In this approach, the decomposition of the image field [18,19], which is a representation of the field by discarding evanescent harmonics, is found by the action of the operator A^{im} [14] on the complex conjugate column of the scattered field E^s, i.e.

$$E^{im} = A^{im}E^{s*}, \tag{8}$$

where the asterisk means complex conjugation and E^{im} is the column containing the expansion coefficients of the image field. Using the A^{im}, one can find the operator that leaves only evanescent

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harmonics [14]:

$$Q = I - 1/2\, A^{im}. \tag{9}$$

By definition of these quantities, the condition $A^{im}Q = 0$ is satisfied. If any of the $Q*$ columns is taken as the column E^s, then it satisfies condition (5) since the action of the operator A^{im} is equal to zero according to Eq. (8). The physical meaning of this condition is that the spatial spectrum of the scattered field contains only evanescent harmonics. For more information about these matrices, see Appendix A.

From here we can give a solution for the first problem. Let $\mathfrak{I}^i$ be a matrix whose columns are the desired fields E^i. Then the following matrix equation is valid:

$$Q^* = T\mathfrak{I}^i. \tag{10}$$

From here it is easy to find the matrix $\mathfrak{I}^i$:

$$\mathfrak{I}^i = T^{-1}Q^*. \tag{11}$$

Let us analyze expression (11). First, expression (11) is valid for an arbitrary particle, for example, for a sphere, ellipsoid, or cylinder, it is only necessary to find the T-matrix in the basis of vector spherical functions. Second, some of the columns of the matrix $\mathfrak{I}^i$ are linearly dependent, since the columns of the matrix Q are linearly dependent, and only half of them are linearly independent [14]. Third, if the matrix T is diagonal in azimuthal modes m, condition (11) can be written separately for each of the azimuthal modes.

Based on the Eq. (10), one can also find a formal solution for the second problem. Let the matrix columns E^i_{inv} contain a set of incident fields, for which the particle must be invisible. Then the matrix T of this particle takes the form:

$$T = Q^*(E^i_{inv})^{-1}. \tag{12}$$

For a complete solution of the problem, it is also necessary to find a particle that has T-matrix (12). It is not at all a fact that such a particle exists, besides it is homogeneous. In fact, it is necessary to make a number of remarks, especially about the dimensions of the matrices E^i_{inv} and Q, how they should be chosen. Here we restrict ourselves to the formal solution (12), we will discuss this problem in more detail in the section 5.

4. Excitation of invisibility states in dielectric spheres

Consider the case of a spherical particle, in this case the T-matrix is easy to find using the Mie theory [20]:

$$T = \begin{pmatrix} t^{\nu\mu}_{lm} & 0 \\ 0 & r^{\nu\mu}_{lm} \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned} t^{\nu\mu}_{lm} &= \delta_{l\nu}\delta_{m\mu}\frac{nj_l(q)[nqj_l(nq)]' - \mu nj_l(nq)[qj_l(q)]'}{\mu nj_l(nq)[qh_l(q)]' - nh_l(q)[nqj_l(nq)]'}, \\ r^{\nu\mu}_{lm} &= \delta_{l\nu}\delta_{m\mu}\frac{\mu j_l(q)[nqj_l(nq)]' - n^2 j_l(nq)[qj_l(q)]'}{n^2 j_l(nq)[qh_l(q)]' - \mu h_l(q)[nqj_l(nq)]'}, \end{aligned} \tag{14}$$

where j_l, h_l are the spherical Bessel and Hankel functions of the first kind, respectively, n is relative refractive index of a particle, μ is particle magnetic permeability, $q = 2\pi R/\lambda$, R is the radius of the particle, λ is the wavelength in the environment, δ is the Kronecker delta. Figure 1 shows the calculation results for the case of $q = 25$ and $n = 1.6$. As the field $\mathbf{E}^{\mathrm{inc}}$ we chose the first, fifth, and tenth columns of the $\mathfrak{I}^i$ matrix obtained from the Eq. (11) for the TM type with m = 1.

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The scattering amplitudes **F** in Fig. 1 testify that the particle is indeed invisible. However, a slight radiation towards $z > 0$ is due to finite dimensions of the matrices T and Q in the Eq. (11). In the case of an increase in dimension, the required field $\mathbf{E}^{\mathrm{inc}}$ increases indefinitely in this example. Thus, it is probably still impossible to achieve perfect invisibility.

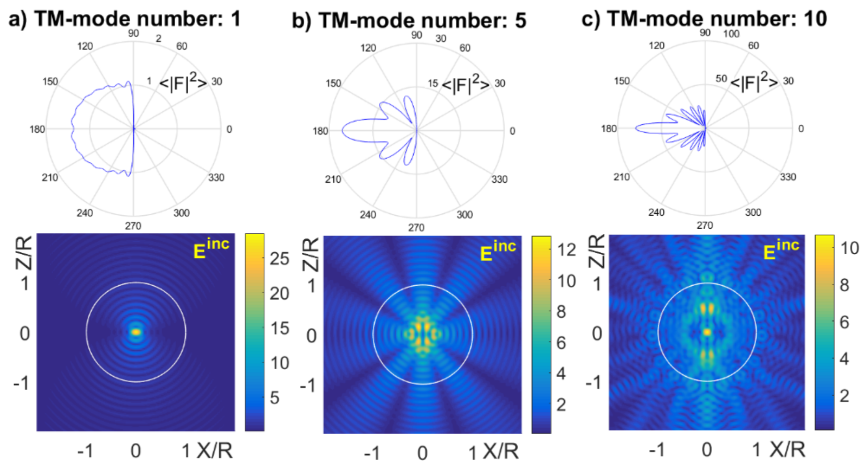

Fig. 1. The φ angle-averaged square of the scattering amplitude $<|\mathbf{F}|^2>$ and the fields $\mathbf{E}^{\mathrm{inc}}$ incident on the field sphere in the plane $y = 0$, calculated by formula (11), for the first - a), fifth - b) and tenth - c) TM modes with $m = 1$.

5. Invisibility particle for a plane wave

We aim to find the shape of a particle with a uniform refractive index, which makes it invisible for a given incident field. Consider the simplest case of an incident plane wave $\mathbf{E}^{inc} = \mathbf{e}_x \exp(-ikz)$, i.e. suppression of backscattering or reflection (recall that we agreed to look for invisible particles in the direction $z > 0$). The scattering geometry is presented in Fig. 2. Note that it was possible to require invisibility for a set of plane waves simultaneously.

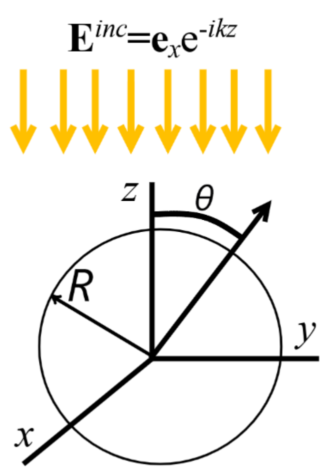

Fig. 2. The scattering geometry. R is the radius of the initial sphere.

The direction of the incident field is not chosen by chance, since the suppression of forward scattering, according to the optical theorem, leads to complete suppression of scattering. In the case of a plane wave in the direction $z > 0$, in our notation, the optical theorem [21] has the form:

$$\sigma_{ext} = -\frac{4\pi}{k^2}\mathrm{Re}(\mathbf{e}_x, \mathbf{F}(\theta = 0)), \tag{15}$$

where σ_{ext} is the extinction cross section [21]. Suppression of forward scattering at $z > 0$, and, in particular, in the direction $\mathbf{F}(\theta = 0)$, leads to the fact that $\sigma_{ext} = 0$. For a non-absorbing particle, and this is our case, the condition $\sigma_{ext} = 0$ means that the particle does not scatter in any directions. This is possible only in the case of $E^s = 0$.

We need the particle to be invisible when it scatters a plane wave, i.e. the scattered field is a linear combination of the columns of the matrix Q^*:

$$E^s = Q^* E^Q, \tag{16}$$

where E^Q is an arbitrary column. In order to satisfy condition (16) we vary the shape of the particle with a fixed refractive index starting from a spherical one. The column E^Q is selected in such a way as to minimize the difference $|E^s - Q^* E^Q|$. In this case $E^Q = Q^{*\dagger} E^s$, where $Q^{*\dagger}$ is a pseudoinverse of Q^*. The need to consider a pseudoinverse matrix arises from the dimension reduction in Q (see Appendix A). In other words, our task is to minimize the value:

$$|(I - QQ^\dagger)^* E^s| \to 0, \tag{17}$$

by varying the shape of the particle. To deal with expression (17), it is necessary to calculate the column E^s, which is the solution of the scattering problem. For this purpose, we use the method of variation of the surface of a particle relative to a spherical surface [22]. The solution is a perturbation theory based on perturbations $f(\theta)$ of the particle surface, i.e. the particle surface is represented as

$$r(\theta) = R(1 + f(\theta)). \tag{18}$$

We propose to look for this perturbation depending only on θ, i.e. the particle is supposed to be symmetrical with respect to rotation about the z-axis. We slightly modified the original method in such a way as to consider perturbations relative to an arbitrary initial surface, this necessity arose to calculate significant deformations of the particle. Under strong deformations relative to a spherical surface, the perturbation theory [22] diverges. In addition, this approach provides greater accuracy in the calculations. We wrote more about our modification of the method in Appendix B.

To minimize (17) we used the numerical method of finding the minimum of the «fminsearch» function presented in the Matlab software. The search for the minimum occurs in an iterative way. In general, with a decrease in the norm of the difference $|(I - QQ^\dagger)^* E^s|$ we observed a decrease in the $\mathbf{F}(\theta, \varphi)$ in comparison with the initial value. Expression (17) makes it possible to suppress $\mathbf{F}$ more effectively than directly minimizing the value of $|\mathbf{F}|$. This fact convincingly testifies to the performance of the proposed method.

As initial particle we considered sphere with the following parameters $2\pi R/\lambda = 3$, $n = 3.4238$. The comparison of forms of initial and deformed particles and its scattering amplitudes is presented in Fig. 3 (see Appendix C for more details).

Based on the results presented in Fig. 3, we see that relatively small deformations lead to the suppression of the scattering indicatrix, the ratio of backscattering amplitudes is approximately *1/25*. Note that the presented result does not indicate the impossibility of further suppression of scattering, this requires more iterations, as well as an increase in the accuracy of the calculation of the E^s. We have also calculated the optical image [19] by collecting the far field from the half-space and restored it in backward propagation for an initial sphere and the deformed particle, see Fig. 4. The image of deformed particle has significantly less intensity but not zero unlike perfect invisible particle.

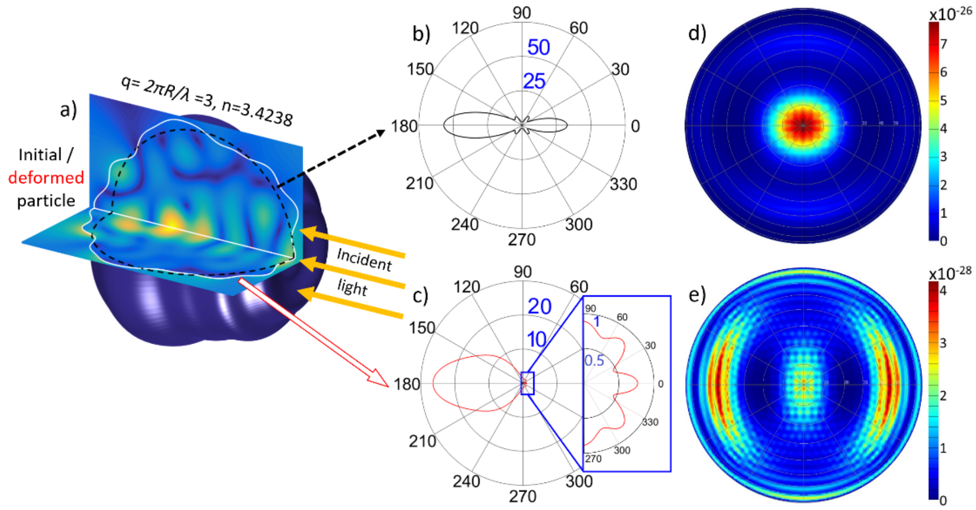

Fig. 3. a) 3D view of the deformed particle and the geometry of a light incidence direction. The insets show the distribution of the field inside the deformed particle. The white solid curve is the shape of the deformed particle, the dotted curve marks the initial spherical shape. b, c) The square of the modulus of the scattering amplitude $<|\mathbf{F}|^2>$ (6) for spherical and deformed particles, accordingly. The backward scattering is significantly suppressed, the ratio of the backscattering amplitudes is about *1/25*. d, e) Lumerical FDTD calculations of the far-field for sphere and deformed particles within $\theta \le \pi/2$.

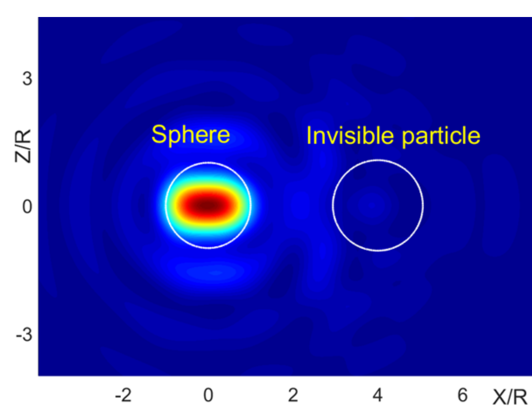

Fig. 4. The optical image for an initial sphere and the deformed particle. The particles are placed along the x-axis. The image is obtained in reflection mode.

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6. Conclusion

The concept of invisible states is proposed; on its basis, the fields exciting these states in spheroidal dielectric particles are found. The proposed approach can be easily generalized to the case of arbitrary particles shape. The method that allows one to calculate the scattering by a particle of a complex shape with significant deviations from a sphere is presented. The approach has been developed for finding invisible particles with a relative backscattering suppression of about *1/25* by varying the particle shape. The considered concept of invisible states still needs to be improved, it is necessary to study the problem for large particles (q*~10-100*), to propose an algorithm for finding initial approximations for the parameters of a variable particle. However, the presented results demonstrate the fundamental performance of the proposed concept of invisible states.

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Data availability. No data were generated or analyzed in the presented research.

Supplemental document. See Supplement 1 for supporting content.

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