

Third-harmonic generation driven by nonlinear anisotropy in asymmetric silicon nanodisks

Cite as: APL Photon. 11, 100801 (2026); doi: 10.1063/5.0335972

Submitted: 27 March 2026 • Accepted: 28 July 2026 •

Published Online: 7 October 2026



A. A. Nazarenko,^{1,2} A. A. Popkova,^{1,2} A. I. Musorin,^{1,2} A. S. Shorokhov,² and A. A. Fedyanin^{2,a)}

AFFILIATIONS

¹ Department of Materials Science, Shenzhen MSU-BIT University, Shenzhen 517182, China

² Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia

Note: This paper is part of the Special Topic on Metasurfaces and Nanoantennas for Nonlinear and Quantum Applications.

a) Author to whom correspondence should be addressed: fedyanin@nanolab.phys.msu.ru

ABSTRACT

Nonlinear asymmetric generation of light in metasurfaces offers a powerful route to compact devices for optical isolation and directional control of harmonic emission without external magnetic bias. In this work, we experimentally demonstrate strongly asymmetric third-harmonic generation in a nonlinear metasurface, which is based on a single-material design and fabricated on a silicon-on-insulator platform (SOI). The metasurface consists of asymmetric silicon nanodisks whose broken mirror symmetry induces nonlinear anisotropy and a direction-dependent cubic nonlinear response. Inverting the pump propagation direction results in a 24-fold modulation of the third-harmonic signal. This pronounced asymmetry arises from the combined effect of the nanodisk structural asymmetry and interference within the finite-thick SiO₂ layer of the SOI stack. Tuning the oxide-layer thickness enables control over the magnitude and spectral position of the asymmetric nonlinear response, making such metasurfaces attractive for applications in compact lasers and optical-signal processing devices.

© 2026 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>). <https://doi.org/10.1063/5.0335972>

I. INTRODUCTION

Metasurfaces appear to be a promising platform for compact photonic devices. They enable precise control over different properties of light and light-matter interaction enhancement due to their resonant nature.^{1–4} Metasurfaces composed of materials with significant nonlinear susceptibility values can be utilized for the enhancement of various nonlinear optical effects,^{5,6} such as harmonic generation.^{7–17} An important range of functionalities is enabled by nonreciprocal metasurfaces. Nonreciprocal metasurfaces exhibit asymmetric transmission of light concerning the interchange of positions between emitters and receivers. Optical nonreciprocity can be achieved through several approaches, including the use of materials with asymmetric permittivity and permeability tensors, such as magneto-optical materials,^{18–21} or the implementation of time-varying systems.^{22–24} However, both strategies encounter difficulties when it comes to nanoscale miniaturization due to the low power efficiencies of time-variant systems or the substantial sizes of components in ferrite-based structures. Therefore, a different approach based on nonlinear light-matter interaction is employed to break reciprocity at the nanoscale with

nonlinear nonreciprocal metasurfaces, where nonreciprocity is induced by such an interaction.^{25,26} At the same time, there exists a conceptually related yet physically distinct class of metasurfaces that do not break reciprocity at the pump wavelength but exhibit a pronounced asymmetry in their nonlinear response. One prominent manifestation of this asymmetry is direction-dependent harmonic generation, where the efficiency of frequency conversion differs depending on the illumination direction.^{27–30} Such a direction-dependent nonlinear generation has been demonstrated in both plasmonic^{28–30} and dielectric²⁷ metasurfaces, but the dielectric ones are more attractive due to their low ohmic losses and their ability to support both electric and magnetic Mie resonances.³¹ Varying the size, shape, and material constants of constituent nanoparticles of dielectric metasurfaces makes it possible to tune electric and magnetic resonances independently^{32,33} and modify the scattering diagram, thus providing an opportunity to achieve asymmetric response. However, the existing metasurface designs typically comprise multiple materials and are challenging to manufacture.^{25,27–29}

In this paper, the metasurface under study consists of metaatoms made of a single material (silicon) with mirror symmetry

broken solely by geometry. We experimentally demonstrate asymmetric third-harmonic generation (THG) in this type of structure. Since the metasurface is fabricated on a silicon-on-insulator (SOI) platform, the resulting asymmetry arises from the combined impact of both the nanoresonator's structural asymmetry and the interference effects within the SiO_2 layer in SOI, which, in turn, allows one to tailor the THG asymmetry by tuning its thickness.

II. RESULTS

The studied metasurface is composed of periodically arranged silicon nanodisks. The periods of the sample are 715 nm in lateral (X and Y) directions [see Fig. 1(a)], and the disks have a diameter of 585 nm and a height of 220 nm. Broken mirror symmetry is introduced into the disk by etching a void from the top side. The depth of the void is 110 nm, which is half of the disk's total height. The array is fabricated by means of a two-step etching process on a silicon-on-insulator (SOI) platform. After etching, the SiO_2 layer is deposited over the entire structure to match the refractive indices of the media above and below the nanodisk array, thereby embedding the array within the silicon dioxide layer (see the [supplementary material](#), I). The overall thickness of the SiO_2 layer is about $3.5\ \mu\text{m}$. A schematic picture of the unit cell and an electron microscopy image of the metasurface before the SiO_2 deposition are presented in Figs. 1(a) and 1(b). The calculated transmittance spectra of the sample shown in Fig. 1(c) indicate that its linear response is independent of the source position, which is required by reciprocity. Figure 1(c) also shows that the metasurface has resonant features (whose nature is explained below) in the near-IR spectral region, at around 1270 and 1390 nm [see Fig. 1(c)]. The sample is expected to exhibit considerable third-harmonic (TH) response in the vicinity of these resonances, since bulk silicon possesses a high third-order nonlinear susceptibility in this range.¹¹ The structure is transparent to pump radiation in this spectral region (1200–1500 nm) but is

opaque to the generated TH light (400–500 nm) from one side due to the strong absorption in the thick silicon substrate ($k \approx 0.05\text{--}0.3^{34}$), which suppresses TH signal propagation in that direction. Consequently, when the source is located above the structure (forward illumination), the nonlinear signal can only be detected in the reflection direction, since it is absorbed by the silicon substrate in the transmission direction. On the contrary, when the source is below the structure (backward illumination), the TH signal is observable only in the transmission direction, since any signal traveling in the reflection direction is absorbed by the substrate in this case. Given these limitations, only two illumination schemes are feasible, and they are presented in Fig. 1(d).

The linear response of the sample is also measured for each of the schemes: reflectance for forward illumination [Fig. 2(a)] and transmittance for backward illumination [Fig. 2(b)]. The optical spectroscopy experiments are conducted using a supercontinuum light source with spectrally narrow pulses selected via an acousto-optic filter and then tuned over the spectral range of interest (the detailed description is provided in the [supplementary material](#), II). The obtained spectra [black dots in Figs. 2(a) and 2(b)] are normalized to either the transmittance spectrum of the glass or the reflectance spectrum of the SOI system without the nanodisk array to account for Fabry–Perot resonances. The normalized spectra indicate the presence of the metasurface resonances at 1270 and 1390 nm, which is consistent with finite element method (FEM) simulations [red curves in Figs. 2(a) and 2(b)] performed in COMSOL Multiphysics (detailed in the [supplementary material](#), III). The deviations between the experimental and numerical results are attributed to the experimental Gaussian beam profile, in contrast to the monochromatic source in simulations, and fabrication imperfections (for example, diameter variations of disks and holes in the array or uneven deposition of SiO_x on top of the metasurface). Analysis of the Cartesian multipole contributions in the reflectance spectra of the nanodisk array embedded in isotropic SiO_2 [see the

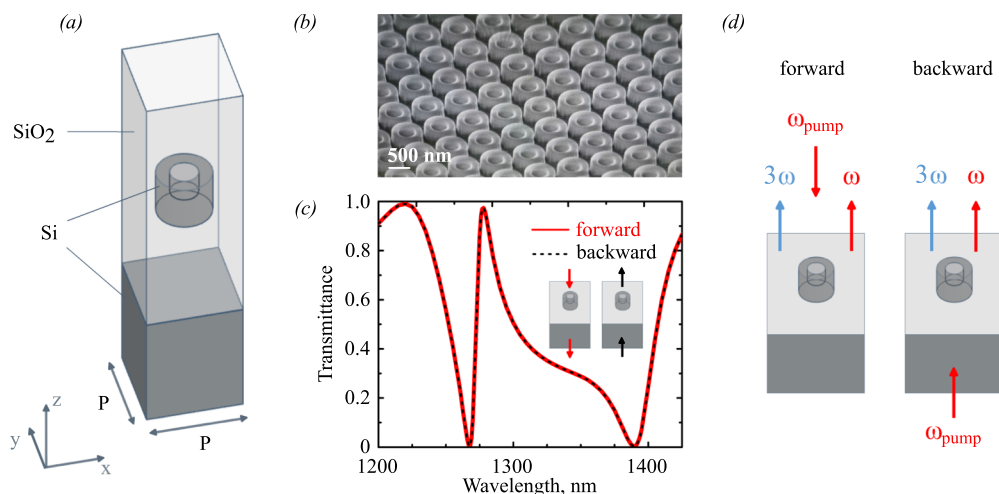


FIG. 1. (a) Schematic illustration of the metasurface unit cell. (b) Scanning electron microscopy image of the metasurface before the deposition of the passivating SiO_2 layer. (c) The calculated linear transmittance spectra for incident radiation falling on the structure from the SiO_2 layer (forward) and from bulk Si (backward). (d) Two experimentally accessible illumination schemes.

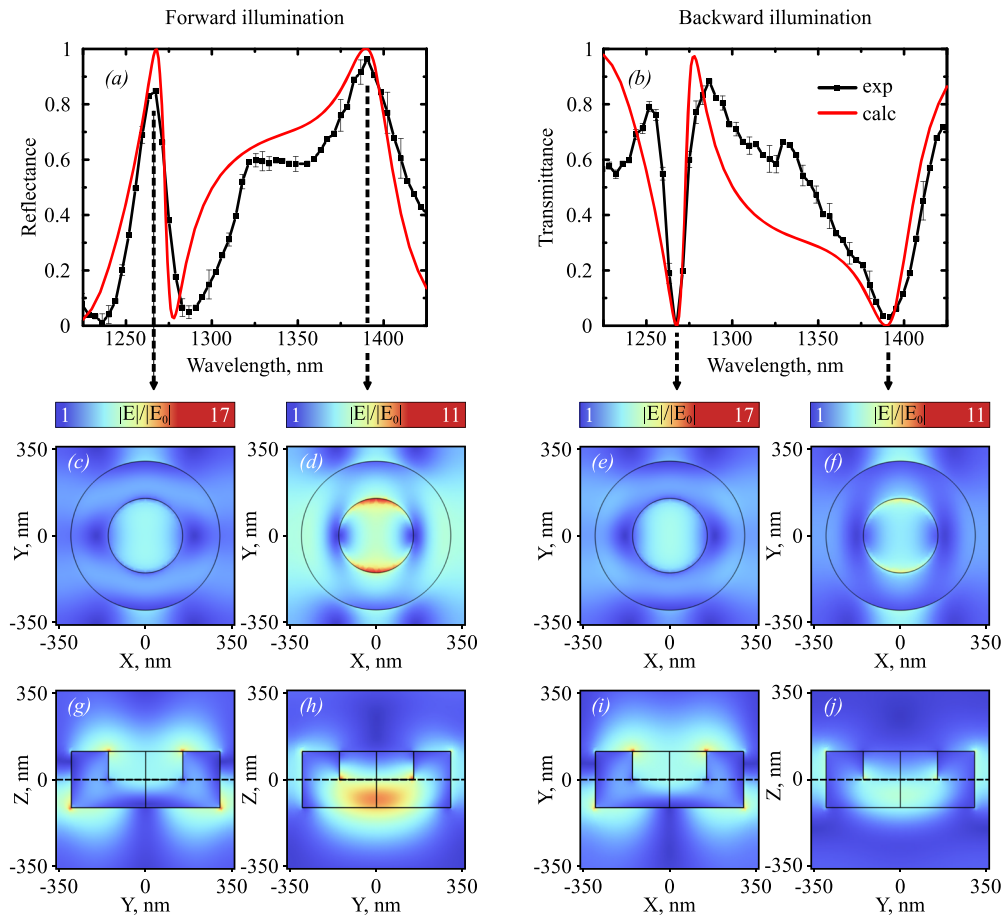


FIG. 2. Linear reflectance (a) and transmittance (b) spectra of the metasurface. Black curves and dots represent experimental data, and red curves refer to numerical calculations. (c)–(j) Electric field distributions inside a unit cell of the metasurface at the short-wave (c), (g), (e), and (i) and long-wave (d), (h), (f), and (j) resonances. Images (c), (d), (g), and (h) correspond to forward illumination, and images (e), (f), (i), and (j) correspond to backward illumination. Cross sections (c), (d), (e), and (f) are made at the middle of the disk's height.

supplementary material, IV, Figs. S1(b) and S1(d)] reveals that the short-wave resonance is mainly a combination of magnetic dipole (MD) and electric quadrupole (EQ) modes, while the long-wave resonance is governed by the electric dipole (ED) and magnetic dipole (MD). Implementing the array within a finite SiO₂ layer on a silicon substrate introduces additional interference effects. The spectral positions of the resonances remain unchanged [see Figs. 2(a), 2(b), S1(a), and S1(c)], and the local field distributions are qualitatively similar in both cases [see Figs. 2(c)–2(j) and S7]. This suggests that the resonances retain their nature, but the finite oxide-layer introduces line shape distortion, manifesting as an additional resonant feature near 1325 nm in Figs. 2(a) and 2(b). The quality factors of the two main resonances at 1270 and 1390 nm are obtained using a Fano model. The shorter-wavelength resonance exhibits $Q \approx 62$ in the calculated spectra and $Q \approx 68$ in the measured spectra, whereas the longer-wavelength resonance shows $Q \approx 37$ in calculations and $Q \approx 30$ experimentally.

The THG spectroscopy results (Fig. 3) demonstrate that the nonlinear response of the sample is asymmetric and strongly depends on the incident radiation direction. It is measured with a spectrally tunable Ti:sapphire laser (pulse duration 150 fs, repetition rate 80 MHz) and an optical parametric oscillator (OPO), which converts pump radiation into the near-IR range (the experimental setup details are provided in the supplementary material, V). The cubic power dependence and the Gaussian spectral profile of the nonlinear signal, matching that of the pump, confirm that the signal originates from third-harmonic generation without any luminescence contribution (see the supplementary material, VI). We also estimated the impact of two-photon-absorption-related free carriers and nonlinear refraction and found their contribution to be negligible compared with THG (see the supplementary material, VII). The obtained TH power spectra are normalized to the TH signal of the silicon substrate of the SOI. The thickness of the silicon substrate in the simulation was 200 nm, which is approximately equal to an

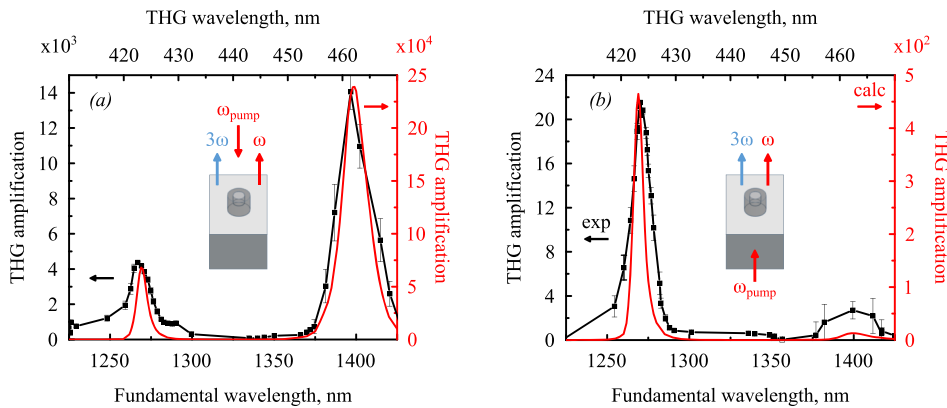


FIG. 3. TH power spectra of the metasurface, normalized to the TH signal of the silicon layer in the SOI structure without the metasurface (THG amplification). Spectra (a) and (b) correspond to forward and backward illumination, respectively. Black curves and dots represent experimental data, and red curves refer to numerical calculations. The insets illustrate two experimentally accessible illumination schemes.

integer number of coherent lengths for the third harmonic generation in both forward and backward directions. This allows us to estimate the signal from a thick substrate, a full-size calculation of which would require too much computing power. These normalized values, referred to as the THG amplification, are represented in Figs. 3(a) and 3(b) by black dots. The spectra exhibit distinct resonances at 1270 and 1400 nm and align well with corresponding numerical calculations performed in COMSOL Multiphysics software. We simulate the nonlinear properties of silicon using External Current Density domains in a form that is consistent with existing literature^{11,35} (see the [supplementary material](#), VIII). We also take into account the fact that the metasurface is not sub-diffractive at the third-harmonic wavelengths ($400\text{--}500\text{ nm}$, $p_x = p_y = 715\text{ nm}$), so multiple diffraction orders can propagate in the surrounding media. Simulations collect all of them, as the TH power is obtained by integrating the normal component of the time-averaged Poynting vector over the upper part of the periodic unit cell. However, the experimental objective with a numerical aperture of $\text{NA} = 0.4$ captures only the zeroth order. Thus, the calculated spectra in Figs. 3 and 4 are normalized by semi-analytical factors (see the [supplementary material](#), IX) to account for the missing power carried by higher diffraction orders. Both experimental and numerical spectra reveal that forward illumination significantly enhances the nonlinear signal at the long-wave resonance compared to the backward one. The

signal increases by a factor of 5×10^3 experimentally and by 2×10^4 numerically. In the case of forward illumination, the signal at the long-wave resonance is greater than at the short-wave resonance, while the opposite is true for the backward case. Furthermore, forward excitation results in the maximum THG amplification of about 1.4×10^4 in experimental data and 2.4×10^5 in simulations compared to the signal of the SOI without the nanodisk array. The discrepancies between the experimental and calculated spectra may be attributed to several factors. They include the finite spectral width of the experimental beam and its angular divergence, which influences the metasurface's response more significantly than that of the silicon substrate, as well as the finite spot size of illumination. Another potential source of mismatch is that the silicon in the SOI structure may be partially modified during the etching process.

The observed asymmetry in the nonlinear response arises from a combination of factors stemming from the complex design of the structure. First, the asymmetric disk geometry itself supports different local field distributions depending on the illumination direction. The field distributions in a unit cell of the nanodisk array in a homogeneous silicon dioxide medium, as well as the analysis of the Cartesian multipole contributions in the reflectance spectra, are presented in the [supplementary material](#), IV and X, and prove the statement. Another source of asymmetry stems from the presence of the finite-thick SiO_2 layer in the SOI-based metasurface.

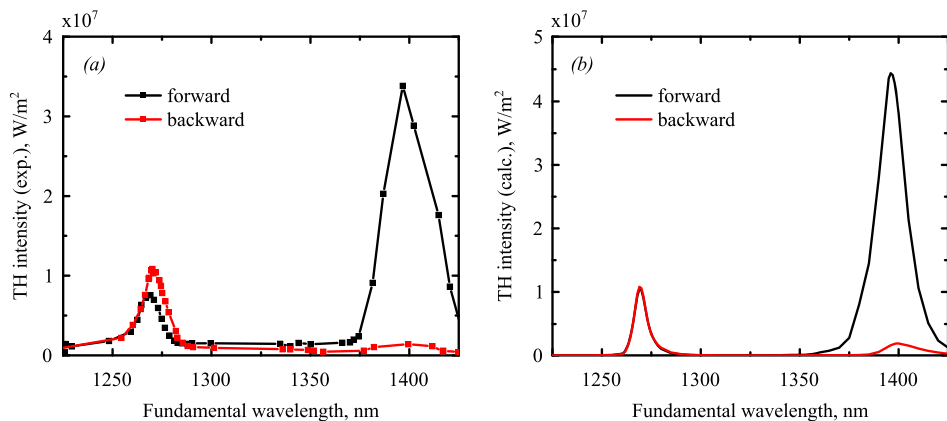


FIG. 4. Experimental (a) and calculated (b) TH intensity spectra of the metasurface. Black curves and dots correspond to forward illumination, and red curves and dots correspond to backward illumination.

Interference of light in the layer occurs both at pump and third-harmonic wavelengths with SiO₂–Si interface contributing the most due to the values of refractive indices in the studied spectral range: $n_{\text{Si}}(1300 \text{ nm}) \approx 3.5$,³⁴ $n_{\text{Si}}(430 \text{ nm}) \approx 4.9$,³⁴ $n_{\text{SiO}_2}(1300 \text{ nm}) \approx 1.45$,³⁶ and $n_{\text{SiO}_2}(430 \text{ nm}) \approx 1.47$.³⁶ In addition, relatively low absorption of silicon in the TH spectral range ($k \approx 0.05$ – 0.3 ³⁴ at 400–500 nm) and the subwavelength thickness of the disk (220 nm) allow the generated TH field itself to be distributed within the structure rather than be absorbed, with the distribution depending on illumination direction. Thus, the overall THG response is a superposition of the aforementioned effects, which are described in more detail in the [supplementary material](#), X.

We employ numerical modeling using FEM, which inherently incorporates these phenomena. The field distributions of the SOI-based metasurface unit cell calculated via this method are presented in [Figs. 2\(c\)–2\(j\)](#). The distributions at the short-wave resonance appear to be nearly identical for different source positions [see [Figs. 2\(c\)](#) and [2\(e\)](#), [2\(g\)](#) and [2\(i\)](#)] and result in minor variations of the nonlinear signal at this resonance, which is illustrated by both experimental [[Fig. 4\(a\)](#)] and numerical [[Fig. 4\(b\)](#)] TH intensity spectra. In contrast, for the long-wave resonance, there is an increase in the TH intensity by ~ 24 times in experimental data and in calculations when the structure is irradiated from above ([Fig. 4](#)), which is accompanied by a considerable enhancement of local fields within the disk [[Figs. 2\(d\)–2\(h\)](#)]. We use intensity for comparison, as it accounts for the illumination spot size, incident power density, and signal acquisition time, ensuring consistency between experiment and simulation. The forward-to-backward ratio of the TH intensity reaches a maximum value of 33 experimentally, whereas the simulations yield a value of 120 [see [Fig. 5\(a\)](#)]. The nonlinear conversion efficiency, which is commonly defined as the ratio of the average powers ($P_{3\omega}/P_{\omega}$), is evaluated for each resonance under forward and backward excitation, with both experimental and calculated values provided in [Table I](#). The most effective conversion is achieved for the long-wave resonance, when the structure is excited from above. The average experimental values on the order of 10^{-7} correlate well with third-harmonic generation results reported in the literature, for

instance, 10^{-7} – 10^{-6} for metasurfaces based on amorphous silicon and silicon nitride nanocylinders,²⁷ as well as 10^{-7} for silicon nanodisks,¹¹ although higher values of conversion efficiency reaching 10^{-5} can also be achieved.³⁷

To further extend our investigation, we examine the impact of the SiO₂ layer thickness d on the THG asymmetry. We numerically vary this parameter while placing the nanodisk array at the center of the oxide layer. Although in the actual metasurface studied in this paper, the array is not located in the center due to fabrication reasons, the simplified symmetric geometry allows us to attribute the observed behavior directly to the oxide thickness without introducing additional positional interference effects. The chosen SiO₂ thickness values correspond to different combinations of integer and half-integer multiples of the long-wave resonance wavelengths in the oxide for both the pump L_{ω} and the third harmonic $L_{3\omega}$, and the corresponding forward-to-backward TH power ratio spectra are shown in [Fig. 5\(b\)](#). The forward-to-backward maximum ratio is achieved for thicknesses satisfying the constructive interference condition for the generated TH wave propagating toward air,

$$d = \left(m + \frac{1}{2}\right)L_{3\omega},$$

namely, for the green ($d \approx 12.5L_{3\omega}$, $m \approx 12$), cyan ($d \approx 10.5L_{3\omega}$, $m \approx 10$), and pink ($d \approx 6.6L_{3\omega}$, $m \approx 6$) curves. In contrast, the blue ($d \approx 11.0L_{3\omega}$, $m \approx 11$), red ($d \approx 12.0L_{3\omega}$, $m \approx 12$), and yellow ($d \approx 8.8L_{3\omega}$, $m \approx 9$) curves correspond to thicknesses close to $d = mL_{3\omega}$, for which destructive interference occurs.

This behavior can be understood by noting that the THG emission of the nanodisks is intrinsically asymmetric, being preferentially directed along the pump propagation direction. For forward illumination (pump propagating downward), the TH field is predominantly emitted toward the substrate, while only a weaker component is emitted toward air. The downward-emitted field is partially reflected at the SiO₂–Si interface and, under the constructive interference condition, adds coherently to the directly upward-emitted field, leading to a strong enhancement of the detected signal. For backward illumination, the situation is reversed: the TH field is

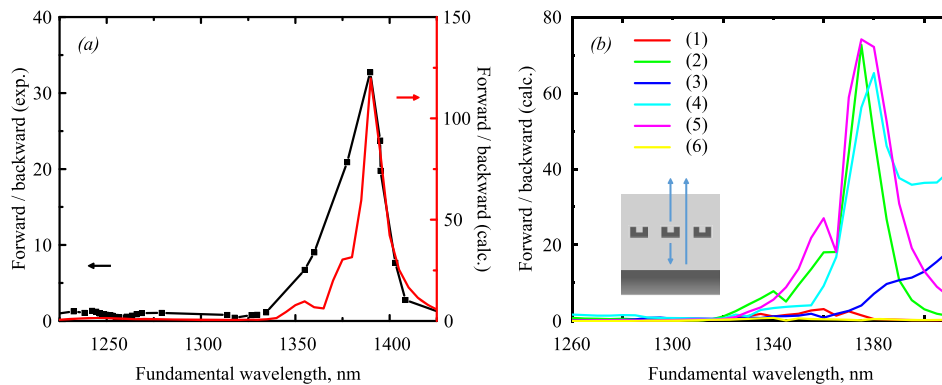


FIG. 5. (a) Forward-to-backward ratio of the TH intensity of the SOI-based metasurface. Black lines and curves correspond to experimental data. Red lines correspond to numerical data. (b) The ratio of the calculated TH power spectra under forward and backward illumination of the SOI-based metasurfaces with varying SiO₂ layer thickness d . The red curve corresponds to $d \approx 3.95L_{\omega} \approx 12.02L_{3\omega}$ (1), the green curve corresponds to $d \approx 4.11L_{\omega} \approx 12.50L_{3\omega}$ (2), the blue curve corresponds to $d \approx 3.62L_{\omega} \approx 11.00L_{3\omega}$ (3), the cyan curve corresponds to $d \approx 3.46L_{\omega} \approx 10.53L_{3\omega}$ (4), the pink curve corresponds to $d \approx 2.17L_{\omega} \approx 6.59L_{3\omega}$ (5), and the yellow curve corresponds to $d \approx 2.89L_{\omega} \approx 8.79L_{3\omega}$ (6), where L_{ω} is the long-wave resonance pump wavelengths and $L_{3\omega}$ is the long-wave resonance third-harmonic wavelength.

TABLE I. Calculated and experimental third-harmonic conversion efficiency ($P_{3\omega}/P_{\omega}$).

Illumination scheme	Calculations		Experiment	
	Short-wave res.	Long-wave res.	Short-wave res.	Long-wave res.
Forward	1.06×10^{-5}	4.50×10^{-5}	4.36×10^{-7}	1.35×10^{-6}
Backward	1.33×10^{-5}	2.37×10^{-6}	3.88×10^{-7}	1.19×10^{-7}

already predominantly emitted toward air, and the reflected contribution represents only a small correction. As a result, constructive interference produces a much larger relative increase of the forward TH signal than of the backward one, thereby maximizing the forward-to-backward ratio at $d = (m + 1/2)L_{3\omega}$.

At the same time, the thickness-dependent interference of the pump also contributes to the effect. The volume-integrated absolute value of the pump field inside the nanodisk exhibits an oscillatory dependence on d with a period consistent with $\sin(2\pi d/L_{\omega})$, indicating that interference at the pump wavelength in the oxide layer modulates the local field enhancement and hence the THG source (for detailed information, see the [supplementary material](#), XI). The forward-to-backward ratios of these pump field integrals follow the same trend as the corresponding THG ratios for the same oxide thicknesses, confirming that both pump and TH interference jointly contribute to the observed asymmetry. In other words, the maxima in the forward-to-backward THG ratio arise when constructive interference at the third-harmonic frequency coincides with pump field configurations, in which the local pump field inside the nanodisks is near a maximum for forward illumination and near a minimum for backward illumination, so that the directionally emitted THG benefits simultaneously from an enhanced local pump field and constructive recombination of the generated third-harmonic field in the direction of air.

III. CONCLUSIONS

In summary, we have experimentally realized a single-material dielectric metasurface that exhibits pronounced asymmetric third-harmonic generation. The structure consists of an array of silicon nanodisks with geometrically broken mirror symmetry fabricated on a silicon-on-insulator platform. Mirror symmetry is broken in each nanodisk by etching a void to half its height. We observe a third-harmonic intensity enhancement of about 10^4 relative to the unpatterned SOI layer and a 24-fold modulation of the nonlinear signal upon reversal of the illumination direction. The SiO_2 layer in the SOI stack introduces additional interference effects, providing an extra degree of freedom to tailor the magnitude and spectral position of the THG asymmetry by adjusting the oxide thickness. The results establish a practical route to passive, bias-free asymmetric nonlinear generation of light compatible with standard silicon photonics fabrication, and they may facilitate the development of compact nonlinear metasurface-based light sources and integrated photonic components, including nonlinear metasurface mirrors that simultaneously provide cavity feedback for semiconductor lasers and directional harmonic generation exclusively out of the laser cavity.

SUPPLEMENTARY MATERIAL

The [supplementary material](#) reports additional information about: sample fabrication process; optical spectroscopy experiment and calculations; the multipole decomposition of the reflection coefficient; different factors contributing to the nature of the THG asymmetry; nonlinear spectroscopy experiment and calculations; diffraction orders of the TH field; power dependence of the TH signal and TH signal spectrum; the impact of free-carrier effects and nonlinear refraction on the THG asymmetry; the role of pump interference in the forward-to-backward TH intensity ratio; and the concept of a nonlinear metasurface mirror for VCSEL.

ACKNOWLEDGMENTS

This work was performed under financial support of the Russian Science Foundation (Grant No. 25-22-00825). The reported study was funded by the Shenzhen Municipal Government and Shenzhen MSU-BIT University. A.A.N. acknowledges the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS” for the scholarship award. The work was carried out using the equipment purchased partly by Moscow State University Development Program. The authors are deeply grateful to Dr. D. A. Smirnova for her valuable advice in developing the initial design in 2021.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

A. A. Nazarenko: Formal analysis (equal); Investigation (equal); Software (equal); Validation (equal); Writing – original draft (equal). **A. A. Popkova:** Data curation (equal); Formal analysis (equal); Investigation (equal); Validation (equal); Writing – original draft (equal). **A. I. Musorin:** Data curation (equal); Formal analysis (equal); Investigation (equal); Project administration (equal); Writing – original draft (equal). **A. S. Shorokhov:** Conceptualization (equal); Formal analysis (equal); Methodology (equal); Supervision (equal); Writing – original draft (equal). **A. A. Fedyanin:** Conceptualization (equal); Funding acquisition (equal); Supervision (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- ¹N. Yu, P. Genevet, M. A. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, and Z. Gaburro, "Light propagation with phase discontinuities: Generalized laws of reflection and refraction," *Science* **334**, 333–337 (2011).
- ²J. Zhang, W. Liu, Z. Zhu, X. Yuan, and S. Qin, "Strong field enhancement and light-matter interactions with all-dielectric metamaterials based on split bar resonators," *Opt. Express* **22**, 30889–30898 (2014).
- ³A. I. Kuznetsov, M. L. Brongersma, J. Yao, M. K. Chen, U. Levy, D. P. Tsai, N. I. Zheludev, A. Faraon, A. Arbabi, N. Yu, D. Chanda, K. B. Crozier, A. V. Kildishev, H. Wang, J. K. W. Yang, J. G. Valentine, P. Genevet, J. A. Fan, O. D. Miller, A. Majumdar, J. E. Fröché, D. Brady, F. Heide, A. Veeraraghavan, N. Engheta, A. Alù, A. Polman, H. A. Atwater, P. Thureja, R. Paniagua-Dominguez, S. T. Ha, A. I. Barreda, J. A. Schuller, I. Staude *et al.*, "Roadmap for optical metasurfaces," *ACS Photonics* **11**, 816–865 (2024).
- ⁴V. V. Yushkov, V. A. Sitnyansky, A. S. Shorokhov, and A. A. Fedyanin, "Analog optical correlation augmented by Mie-resonant metasurfaces," *ACS Photonics* **11**, 2506–2512 (2024).
- ⁵A. Krasnok, M. Tymchenko, and A. Alù, "Nonlinear metasurfaces: A paradigm shift in nonlinear optics," *Mater. Today* **21**, 8–21 (2018).
- ⁶P. Vabishchevich and Y. Kivshar, "Nonlinear photonics with metasurfaces," *Photonics Res.* **11**, B50–B64 (2023).
- ⁷M. W. Klein, C. Enkrich, M. Wegener, and S. Linden, "Second-harmonic generation from magnetic metamaterials," *Science* **313**, 502–504 (2006).
- ⁸S. Liu, M. B. Sinclair, S. Saravi, G. A. Keeler, Y. Yang, J. Reno, G. M. Peake, F. Setzpfandt, I. Staude, T. Pertsch, and I. Brener, "Resonantly enhanced second-harmonic generation using III-V semiconductor all-dielectric metasurfaces," *Nano Lett.* **16**, 5426–5432 (2016).
- ⁹A. A. Popkova, I. M. Antropov, G. I. Tselikov, G. A. Ermolaev, I. Ozerov, R. V. Kirtaev, S. M. Novikov, A. B. Evlyukhin, A. V. Arsenin, V. O. Bessonov, V. S. Volkov, and A. A. Fedyanin, "Nonlinear exciton-Mie coupling in transition metal dichalcogenide nanoresonators," *Laser Photonics Rev.* **16**, 2100604 (2022).
- ¹⁰A. A. Nazarenko, A. M. Chernyak, A. I. Musorin, A. S. Shorokhov, L. Ding, V. Valuckas, M. Nonahal, I. Aharonovich, S. T. Ha, A. I. Kuznetsov, and A. A. Fedyanin, "Cryogenic nonlinear microscopy of high-Q metasurfaces coupled with transition metal dichalcogenide monolayers," *Nanophotonics* **13**, 3429–3436 (2024).
- ¹¹M. R. Shcherbakov, D. N. Neshev, B. Hopkins, A. S. Shorokhov, I. Staude, E. V. Melik-Gaykazyan, M. Decker, A. A. Ezhov, A. E. Miroshnichenko, I. Brener, A. A. Fedyanin, and Y. S. Kivshar, "Enhanced third-harmonic generation in silicon nanoparticles driven by magnetic response," *Nano Lett.* **14**, 6488–6492 (2014).
- ¹²J. Yu, S. Park, I. Hwang, D. Kim, J. Y. Jung, and J. Lee, "Third-harmonic generation from plasmonic metasurfaces coupled to intersubband transitions," *Adv. Opt. Mater.* **7**, 1801510 (2019).
- ¹³D. Smirnova, S. Kruk, D. Leykam, E. Melik-Gaykazyan, D.-Y. Choi, and Y. Kivshar, "Third-harmonic generation in photonic topological metasurfaces," *Phys. Rev. Lett.* **123**, 103901 (2019).
- ¹⁴M. Kroychuk, A. S. Shorokhov, D. F. Yagudin, D. A. Shilkin, D. A. Smirnova, I. Volkovskaya, M. R. Shcherbakov, G. Shvets, and A. A. Fedyanin, "Enhanced nonlinear light generation in oligomers of silicon nanoparticles under vector beam illumination," *Nano Lett.* **20**, 3471–3477 (2020).
- ¹⁵K. I. Okhlopkov, A. Zilli, A. Tognazzi, D. Rocco, L. Fagiani, E. Mafakheri, M. Bollani, M. Finazzi, M. Celebrano, M. R. Shcherbakov, C. De Angelis, and A. A. Fedyanin, "Tailoring third-harmonic diffraction efficiency by hybrid modes in high-Q metasurfaces," *Nano Lett.* **21**, 10438–10445 (2021).
- ¹⁶J. T. Wang, P. Tonkaev, K. Koshelev, F. Lai, S. Kruk, Q. Song, Y. Kivshar, and N. C. Panoui, "Resonantly enhanced second- and third-harmonic generation in dielectric nonlinear metasurfaces," *Opto-Electron. Adv.* **7**, 230186 (2024).
- ¹⁷G. Zograf, K. Koshelev, A. Zalogina, V. Korolev, R. Hollinger, D.-Y. Choi, M. Zuerch, C. Spielmann, B. Luther-Davies, D. Kartashov, S. V. Makarov *et al.*, "High-harmonic generation from resonant dielectric metasurfaces empowered by bound states in the continuum," *ACS Photonics* **9**, 567–574 (2022).
- ¹⁸A. B. Khanikaev, S. H. Mousavi, G. Shvets, and Y. S. Kivshar, "One-way extraordinary optical transmission and nonreciprocal spoof plasmons," *Phys. Rev. Lett.* **105**, 126804 (2010).
- ¹⁹W. Li, Q. Yang, O. You, C. Lu, F. Guan, J. Liu, J. Shi, and S. Zhang, "Magneto-optical chiral metasurfaces for achieving polarization-independent nonreciprocal transmission," *Sci. Adv.* **10**, eadm7458 (2024).
- ²⁰S.-W. Cheong, D. Talbayev, V. Kiryukhin, and A. Saxena, "Broken symmetries, non-reciprocity, and multiferroicity," *npj Quantum Mater.* **3**, 19 (2018).
- ²¹J. Cui and Q. Zhu, "An absorbing material based on the non-reciprocity of magnetic surface plasmon state of ferrites," *Front. Phys.* **20**, 064200 (2025).
- ²²Z. Yu and S. Fan, "Complete optical isolation created by indirect interband photonic transitions," *Nat. Photonics* **3**, 91–94 (2009).
- ²³N. A. Estep, D. L. Sounas, J. Soric, and A. Alù, "Magnetic-free non-reciprocity and isolation based on parametrically modulated coupled-resonator loops," *Nat. Phys.* **10**, 923–927 (2014).
- ²⁴M. H. Mostafa, M. S. Mirmoosa, M. S. Sidorenko, V. S. Asadchy, and S. A. Tretyakov, "Temporal interfaces in complex electromagnetic materials: An overview [Invited]," *Opt. Mater. Express* **14**, 1103–1127 (2024).
- ²⁵A. Tripathi, C. F. Ugwu, V. S. Asadchy, I. Faniayeu, I. Kravchenko, S. Fan, Y. Kivshar *et al.*, "Nanoscale optical nonreciprocity with nonlinear metasurfaces," *Nat. Commun.* **15**, 5077 (2024).
- ²⁶M. Cotrufo, A. Cordaro, D. L. Sounas, A. Polman, and A. Alù, "Passive bias-free non-reciprocal metasurfaces based on thermally nonlinear quasi-bound states in the continuum," *Nat. Photonics* **18**, 81–90 (2024).
- ²⁷S. S. Kruk, L. Wang, B. Sain, Z. Dong, J. Yang, T. Zentgraf, and Y. Kivshar, "Asymmetric parametric generation of images with nonlinear dielectric metasurfaces," *Nat. Photonics* **16**, 561–565 (2022).
- ²⁸D. Rocco, A. Locatelli, L. Carletti, M. A. Vincenti, and C. De Angelis, "Nonlinear asymmetric imaging with AlGaAs metasurface," *Opt. Express* **32**, 11673–11680 (2024).
- ²⁹S. Boroviks, A. Kiselev, K. Achouri, and O. J. F. Martin, "Demonstration of a plasmonic nonlinear pseudodiode," *Nano Lett.* **23**, 3362–3368 (2023).
- ³⁰E. Mobini, R. Alae, R. W. Boyd, and K. Dolgaleva, "Giant asymmetric second-harmonic generation in bianisotropic metasurfaces based on bound states in the continuum," *ACS Photonics* **8**, 3234–3240 (2021).
- ³¹I. Staude and J. Schilling, "Metamaterial-inspired silicon nanophotonics," *Nat. Photonics* **11**, 274–284 (2017).
- ³²A. I. Kuznetsov, A. E. Miroshnichenko, Y. H. Fu, J. Zhang, and B. Luk'yanchuk, "Magnetic light," *Sci. Rep.* **2**, 492 (2012).
- ³³I. Staude, A. E. Miroshnichenko, M. Decker, N. T. Fofang, S. Liu, E. Gonzales, J. Dominguez, T. S. Luk, D. N. Neshev, I. Brener, and Y. Kivshar, "Tailoring directional scattering through magnetic and electric resonances in subwavelength silicon nanodisks," *ACS Nano* **7**, 7824–7832 (2013).
- ³⁴C. Schinke, P. Christian Peest, J. Schmidt, R. Brendel, K. Bothe, M. R. Vogt, I. Kröger, S. Winter, A. Schirmacher, S. Lim, H. T. Nguyen, and D. MacDonald, "Uncertainty analysis for the coefficient of band-to-band absorption of crystalline silicon," *AIP Adv.* **5**, 067168 (2015).
- ³⁵P. D. Maker and R. W. Terhune, "Study of optical effects due to an induced polarization third order in the electric field strength," *Phys. Rev.* **137**, A801–A818 (1965).
- ³⁶I. H. Malitson, "Interspecimen comparison of the refractive index of fused silica," *J. Opt. Soc. Am.* **55**, 1205–1209 (1965).
- ³⁷C. U. Hail, L. Michaeli, and H. A. Atwater, "Third harmonic generation enhancement and wavefront control using a local high-Q metasurface," *Nano Lett.* **24**, 2257–2263 (2024).